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# Mine Operation Research 

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## Mine Operation Research

People, who have been working in general operation research topics, divided different tool box used in their business as follows:

| Simulation | $29 \%$ |
| :--- | :--- |
| Linear Programming | $21 \%$ |
| Network operation | $14 \%$ |
| Inventory theory | $12 \%$ |
| Non- linear programming | $8 \%$ |
| Dynamic programming | $4 \%$ |
| Integrated programming | $3 \%$ |
| Others | $9 \%$ |

These values confirm the fact that the simulation is the most important operation search practice and that is applicable also for mining engineering discipline.

In this course we will touch ground about the formulation and solution approach of the following mining operation tasks:

1. Time motion studies
2. Blending and scheduling
3. Ultimate pit limit (LP)
4. Mine to mill schedule
5. Production scheduling
6. Truck dispatching
7. Cut off optimization (traditional)
8. Cut off optimization (heuristic)
9. Cut off optimization (Lan's approach)

The materials presented herein are excerpts from my records as PhD student for MINE433: Mine System Analysis class offered by Prof. Kadri Dagdelen of the mining engineering department - Colorado school of mines - USA.

## Part 1

## Mining Systems Simulation

Software applicable to this module:

## 

TALPAC is the mining industry's leading haulage and loading simulator and is used by mining companies globally to simulate a truck and loader fleet over a haul route. Using proven logic that models real haulage situations, TALPAC enables users to study the measurable factors that affect productivity, and how fleet will react to them.


HAULSIM connects the fleet assets, mining operational plans and the people to build a 'digital twin' of any mining operation - open pit or underground - to deliver an accurate representation of any mine sites haulage operations.

## Part 1: Mining Systems Simulation

### 1.1. Steps in Simulation Study

The required steps for building up a simulation model are nearly the same for most cases. The following representing a guide line for these steps:

1 Study the system you are going to try to simulate.
2 Time motion study (monitor and record the required time for each step)
3 Complete Data Collection
4 Data Analysis Procedure.
5 Develop a computer program (according to the language of the simulation)
6 Analysis of the program results (validate the model to see how accurate is it)
7 Sensitivity of the program for various conditions
8 Presentation of the results.

### 1.2. Critical Elements in simulation

The most critical element in simulation is the determination of the probability distribution of each studied phenomena. Also the generation Random Realization from the probability distribution we obtained

## Example

Truck- Shovel- Dump Model is being studied the following are the recorded durations results from monitoring of truck travel times on this system:

| Travel time <br> (Min) | No of <br> Observation |
| :---: | :---: |
| 6 | 40 |
| 7 | 60 |
| 8 | 90 |
| 9 | 150 |
| 10 | 150 |
| 11 | 150 |
| 12 | 160 |
| 13 | 80 |
| 14 | 70 |
| 15 | 50 |



Calculate: the possible value for the travel time of that track system

## Answer

The following table representing the cumulative probability distribution of each observed travel time

Table(1)

| Travel time <br> (Min) <br> (I) | No of <br> Observation <br> (II) | Probability <br> Of observation <br> (III) $=(\mathbf{I I}) / \mathbf{1 0 0 0}$ | Cumulative <br> probability |
| :---: | :---: | :---: | :---: |
| 6 | 40 | 0.04 | 0.04 |
| 7 | 60 | 0.06 | 0.10 |
| 8 | 90 | 0.09 | 0.19 |
| 9 | 150 | 0.15 | 0.34 |
| 10 | 150 | 0.15 | 0.49 |
| 11 | 150 | 0.15 | 0.64 |
| 12 | 160 | 0.16 | 0.80 |
| 13 | 80 | 0.08 | 0.88 |
| 14 | 70 | 0.07 | 0.95 |
| 5 | 50 | 0.07 | 1.00 |

Total Observations $=1000$

Now we can readjust this table for a traveling time ranges with corresponding frequencies.

Table (2)

| Travel time <br> range <br> (I) | No of <br> Observation <br> (II) | Probability <br> Of observation <br> (III) =(II)/1000 | Cumulative <br> probability |
| :---: | :---: | :---: | :---: |
| $5.5: 6.5$ | 40 | 0.04 | 0.04 |
| $6.5: 7.5$ | 60 | 0.06 | 0.10 |
| $7.5: 8.5$ | 90 | 0.09 | 0.19 |
| $8.5: 9.5$ | 150 | 0.15 | 0.34 |
| $9.5: 10.5$ | 150 | 0.15 | 0.49 |
| $10.5: 11.5$ | 150 | 0.15 | 0.64 |
| $12.5: 13.5$ | 160 | 0.16 | 0.80 |
| $13.5: 14.5$ | 80 | 0.08 | 0.88 |
| $14.5:: 15.5$ | 70 | 0.07 | 0.95 |
| $15.5: 16.5$ | 50 | 0.07 | 1.00 |



According to the previous figure, there should be a certain random number that actually representing travel time of the studied truck system. The method to produce that Number is using the so-called Random Number Generating Equations. The following will illustrate one of the various generating equations.

### 1.3. Random Number generator

$$
\begin{aligned}
& \mathbf{n}_{(\mathbf{i}+1)}=k n_{(i)}(\bmod m) \\
& \mathbf{r}_{(\mathbf{i})}=\mathbf{n}_{(\mathbf{i})}(\bmod 100) / 100
\end{aligned}
$$

where :

$$
\mathrm{n}(\mathrm{i})=\text { seed }
$$

$\mathrm{K}=$ given constant
$\mathrm{r}_{(\mathrm{i})}=$ Random number representing the cumulative probability. $m=$ given number for a certain random number from 0:97

Given the values of the following: $K=3, n_{(0)}=9, m=97$

$$
\begin{aligned}
& \mathbf{n}_{(1)}=k \mathbf{n}_{(0)}(\bmod \mathbf{m}) \\
& \mathbf{n}_{(1)}=3 * 9 *(\bmod 97)=27 \\
& =27-(0 * 97)=27 \\
& \mathbf{r}_{(1)}=27(\bmod 100) / 100=0.27 \\
& =27-(0 * 100) / 100=0.27 \\
& \mathbf{n}_{(\mathbf{2})}=\mathbf{k ~ n}_{(\mathbf{1})}(\bmod \mathbf{m}) \\
& n_{(2)}=3 * 27 *(\bmod 97)=81 \\
& \text { 81-(0*97) = } 81 \\
& \mathbf{r}_{(2)}=81(\bmod 100) / 100=0.81 \\
& \mathbf{n}_{(3)}=\mathbf{k} \mathbf{n}_{(\mathbf{2})}(\bmod \mathbf{m}) \\
& n_{(3)}=3 * 81 *(\bmod 97)=49 \\
& \text { 243-( } 2 * 97 \text { ) }=49 \\
& \mathbf{r}_{(3)}=49(\bmod 100) / 100=0.49
\end{aligned}
$$

## To calculate $(\bmod m)$

Practice this,
$13(\bmod 10)=13-1 * 10=3$
$23(\bmod 10)=23-2 * 10=3$
$33(\bmod 10)=33-3 * 10=3$
$43(\bmod 10)=43-4 * 10=3$
$3(\bmod 10)=3-0 * 10=3$

To calculate $\mathbf{n}_{(1)}$
$27(\bmod 97)=27-0 * 97=27$

To calculate $\mathbf{n}_{(1)}$
$3 * 81(\bmod 97)=243-2 * 97=27$

Continuing with this calculations till reach the value of $\mathbf{r}_{(11)}$, The following table contains the final random numbers produced by the generating equation, and the corresponding travel time for each random number

Table (3)

| $\mathbf{r}_{(\mathbf{i})}$ | Interpolation <br> Range <br> Table (1) | Corresponding <br> time lower <br> range case |
| :---: | :---: | :---: |
| 0.27 | $8: 9$ | 8 |
| 0.81 | $12: 13$ | 12 |
| 0.49 | $10: 11$ | 10 |
| 0.5 | $10: 11$ | 10 |
| 0.53 | $10: 11$ | 10 |
| 0.62 | $11: 12$ | 11 |
| 0.89 | $13: 14$ | 13 |
| 0.73 | $11: 12$ | 11 |
| 0.25 | $8: 9$ | 8 |
| 0.75 | $11: 12$ | 11 |
| 0.31 | $8: 9$ | 8 |

## Example:

Underground mine work using a Continuous Miner, Shuttle Car (CM-SC) and Belt Conveyor (BC) at the end. The following data was collected from the mine representing usual operation representing collected field data:

| Belt conveyor <br> Duration between <br> failures | Cumulative <br> probability | (CM-SC) <br> duration between <br> failures <br> $(\mathbf{H r})$ | Cumulative <br> probability |
| :---: | :---: | :---: | :---: |
| $(\mathbf{H r})$ | 0.1 | $\mathbf{1}$ | 0.2 |
| $\mathbf{4}$ | 0.25 | $\mathbf{2}$ | 0.6 |
| $\mathbf{6}$ | 0.5 | $\mathbf{3}$ | 0.8 |
| $\mathbf{8}$ | 0.75 | $\mathbf{4}$ | 0.88 |
| $\mathbf{1 0}$ | 0.9 | $\mathbf{5}$ | 0.95 |
| $\mathbf{1 2}$ | 1.0 | $\mathbf{6}$ | 1 |
| $\mathbf{1 4}$ |  |  |  |

Note: production rate of the system is $100 \mathrm{tone} / \mathrm{hr}$ and the shift time is 8 hr . If the belt failed it requires at least 3 hr to be fixed. The CM-SC system takes 1 hr to be fixed.
It is required to estimate the actual production of this system in 4 shifts?
Answer
We bring the so-called random reality of the problem as follows:

$$
\begin{gathered}
\mathbf{n}_{(1)}=k \mathbf{n}_{(0)}(\bmod \mathbf{m}) \\
\mathbf{r}_{(\mathrm{i})}=\mathbf{n}_{(\mathrm{i})}(\bmod 100) / \mathbf{1 0 0}
\end{gathered}
$$

Where:

$$
\mathrm{K}=5, \mathrm{~m}=201, \quad \mathrm{n}_{(0)}=3
$$

The following are the calculated values for $\mathbf{r}_{(\mathbf{i})}$ the system:

| Belt conveyor |  |  | $($ CM-SC ) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{i}$ | $\mathbf{r}_{\mathbf{i}}$ | $\mathbf{T}_{\mathbf{i}}$ | $\mathbf{i}$ | $\mathbf{r}_{\mathbf{i}}$ | $\mathbf{T}_{\mathbf{i}}$ |
| 1 | 0.75 | 10 | 1 | 0.15 | 1 |
| 2 | 0.1 | 4 | 2 | 0.75 | 2 |
| 3 | 0.9 | 12 | 3 | 0.74 | 2 |
| 4 | 0.25 | 6 | 4 | 0.66 | 2 |
| 5 | 0.3 | 6 | 5 | 0.29 | 1 |
| 6 |  |  | 6 | 0.42 | 1 |

[^0]

## Diagrammatic Representation of the four shifts

So the number of system in business is equal to the up hour $=16 \mathrm{hr}$ The availability of these system $=16 * 100 / 32=50 \%$
The actual production of that system will be $=16 * 100=1600$ tone $/ 4$ shifts

### 1.4. Different modes Random Numbers generators

Until now, we still suppose that the behavior of the recorded data of any system is following a Norma linear distribution. Which is actually not possible in all cases?
Now we will study the methods of producing Random numbers in case of different modes.

## 1-Statistical summary

$f(x)=$ probability density function
$F(x)=$ Cumulative probability density function

$$
F(x)=p(x \leq a)=\int_{-\infty}^{\infty} f(x) d x
$$

$\mathrm{F}(\mathrm{x})$ representing the probability that $(\mathrm{x})$ is going to take values less than, equal to (a)


$F(x)$ representing the area under the curve of Function $f\left(x_{0}\right)$
Now we can start with variable continuous distributions

1- Normal Distribution (Bell shaped function)

$$
f(x)=\frac{e^{-1 / 2((x-v x) / \sigma x)^{2}}}{\sigma_{x} \sqrt{2 \Pi}}
$$

Where
$v_{x}=$ mean value of the collected data
$\sigma_{x}=$ Standard deviation of the collected data
$-\infty<\mathrm{x}<\infty$
for more normalized distribution, the $\nu_{x}=0$ and $\sigma_{x}=1$
then the $\mathrm{F}(\mathrm{z})=$

$$
f(z)=\frac{e^{-1 / 2\left((z)^{2}\right.}}{\sigma_{x} \sqrt{2 \Pi}}
$$

Then to calculate the important used parameter

$$
z=\frac{x-v x}{\sigma_{x}}
$$

The probability range can be estimated as follows


Use of random numbers with normal displacement


We can use inverse transformation to solve for (x) i.e. $\mathrm{x}=\mathrm{f}-1(\mathrm{r} 0)$ for example

$$
\begin{aligned}
\mathrm{F}(\mathrm{x}) & =2 \mathrm{x} \quad 0<\mathrm{x}<1 \\
& =0 \quad \quad \mathrm{x}<0, \mathrm{x}>1 \\
\mathrm{r} & =F(x)=\int_{0}^{x} 2 x d x \quad \text { so } \mathrm{r}=\mathrm{x} 2 \quad \text { or } \mathrm{x}=\sqrt{ } \mathrm{r}
\end{aligned}
$$

for the normal distribution :

$$
f(x)=\frac{e^{-1 / 2((x-v x) / \sigma x)^{2}}}{\sigma_{x} \sqrt{2 \Pi}}
$$

Then using this equation to generate random variants from the normal density function

$$
x=\sigma_{x}(12 / k)^{1 / 2}\left(\sum_{i=1}^{k} r i-k / 2\right)+v x
$$

Recommended $\mathrm{k}=12$, then use the following equation for producing random numbers

$$
x=\sigma_{x}\left(\sum_{i=1}^{k} r i-6\right)+v x
$$

## Example

Generate normally distributed truck arrival times coming from normal distributed with a mean $v x=13.3$ and standard deviation $\sigma_{x}=1.3$. Use the random number generator given below:
$\mathrm{n}(\mathrm{i}+1)=\mathrm{k} \mathrm{ni}(\bmod \mathrm{m})$ where: $\mathrm{k}=3, \mathrm{no}=9, \mathrm{~m}=97$ and $\mathrm{ri}=\mathrm{ni}(\bmod 100) / 100$

## Answer

Step 1

| Sequence | Random <br> variants |
| :---: | :---: |
| 1 | 0.27 |
| 2 | 0.81 |
| 3 | 0.49 |
| 4 | 0.5 |
| 5 | 0.53 |
| 6 | 0.62 |
| 7 | 0.89 |
| 8 | 0.73 |
| 9 | 0.25 |
| 10 | 0.75 |
| 11 | 0.31 |
| 12 | 0.93 |
| Total | $\mathbf{7 . 0 8}$ |

### 1.5. Mont-Carlo-stochastic simulation

Generate random data according to a certain dissertation.

### 1.5.1. Normal distributed

## Example:

Assume that time between failures for a continuous miner- shuttle car system is normally distributed and the mean $v x$ and standard deviation ox can calculated from the following time study data :

| Time between <br> delays (hours) | Mid point <br> Interval <br> $\left(\mathbf{X}_{\mathbf{i}}\right)$ | Frequency |
| :---: | :---: | :---: |
| $0: 1$ | 0.5 | 0.2 |
| $1-2$ | 1.5 | 0.4 |
| $2-3$ | 2.5 | 0.2 |
| $3-4$ | 3.5 | 0.08 |
| $4-5$ | 4.5 | 0.07 |
| $5-6$ | 5.5 | 0.05 |
| Total=sum(Pi) |  | 1.0 |

We will use the following formulas to produce the value of $\sigma x$ and $v x$ :

$$
\begin{gather*}
v_{x}=\left(\sum_{i=1}^{n} x_{i} * p_{i}\right) / \sum_{i=1}^{x} p_{i}  \tag{1}\\
\sigma_{x}^{2}=\sum_{i=1}^{n}\left(x_{i}-v_{x}\right)^{2} * p_{i} \tag{2}
\end{gather*}
$$

So the value of mean can be calculated as follows:

$$
\begin{aligned}
v x & =(\text { mid point of the each interval })(\text { frequency each }) /(\text { sum }(\mathrm{pi}) \\
& =\frac{0.5 * 0.2+1.5 * 0.4+2.5 * 0.2+3.5 * 0.08+4.5 * 0.07+5.5 * 0.05}{(0.2+0.4+0.2+0.08+0.07+0.05)}=2.07
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{x}^{2}= & (0.5-2.07)^{2 *} 0.2+(1.5-2.07)^{2} * 0.4+(2.5-2.07)^{2 *} 0.2+(3.5-2.07)^{2} * 0.08+ \\
& (4.5-2.07)^{2 *} 0.07+(5.5-2.07)^{2 *} 0.05=1.825
\end{aligned}
$$

$$
\sigma_{x}=1.35
$$

Hence we have a normal distribution data with mean=2.07 and standard deviation=1.35 now we can start generating random numbers from this data using the following formulas:

$$
\begin{aligned}
& \mathbf{n}_{(\mathbf{1})}=\mathbf{k} \mathbf{n}_{(\mathbf{0})}(\bmod \mathbf{m}) \\
& \mathrm{r}_{(\mathrm{i})}=\mathrm{n}_{(\mathrm{i})}(\bmod 100) / 100
\end{aligned}
$$

Where: $\mathrm{K}=5, \mathrm{n}_{(0)}=3, \mathrm{~m}=201$

$$
\begin{aligned}
& \mathrm{n}_{(1)}=5 * 3(\bmod 201)=15 \quad \text { and } \mathrm{ri}=15(\bmod 100) / 100=0.15 \\
& \mathrm{n}_{(2)}=5^{*} 15(\bmod 201)=75 \quad \text { and } \mathrm{ri}=75(\bmod 100) / 100=0.75 \\
& \mathrm{n}_{(3)}=5 * 75(\bmod 201)=174 \quad \text { and } \mathrm{ri}=174(\bmod 100) / 100=0.74 \\
& \mathrm{n}_{(4)}=5^{*} 174(\bmod 201)=66 \quad \text { and } \mathrm{ri}=66(\bmod 100) / 100=0.66 \\
& n_{(5)}=5 * 66(\bmod 201)=129 \quad \text { and } \mathrm{ri}=129(\bmod 100) / 100=0.29 \\
& \mathrm{n}_{(6)}=5^{*} 129(\bmod 201)=42 \quad \text { and } \mathrm{ri}=42(\bmod 100) / 100=0.42 \\
& \mathrm{n}_{(7)}=5 * 42(\bmod 201)=9 \quad \text { and } \mathrm{ri}=9(\bmod 100) / 100=0.90 \\
& \mathrm{n}_{(8)}=5 * 9(\bmod 201)=45 \quad \text { and } \mathrm{ri}=45(\bmod 100) / 100=0.45 \\
& \mathrm{n}_{(9)}=5 * 45(\bmod 201)=24 \quad \text { and } \mathrm{ri}=24(\bmod 100) / 100=0.24 \\
& \mathrm{n}_{(10)}=5 * 24(\bmod 201)=120 \quad \text { and } \mathrm{ri}=129(\bmod 100) / 100=0.20 \\
& \mathrm{n}_{(11)}=5 * 120(\bmod 201)=198 \text { and } \mathrm{ri}=198(\bmod 100) / 100=0.98 \\
& \mathrm{n}_{(12)}=5^{*} 66(\bmod 201)=129 \text { and } \mathrm{ri}=129(\bmod 100) / 100=0.29
\end{aligned}
$$

$$
\Sigma \text { ri }=6.07
$$

As the ( x ) is normally distributed variable, if it is generated according to:

$$
x=\sigma_{x}\left(\sum_{i=1}^{k} r i-6\right)+v x
$$

First arrival time $=\mathrm{X} 1=1.35(6.07-6)+2.07=2.16 \mathrm{hr}$

For additional values of the arrival times you have to generate the Random numbers from (n13 to n24) and carry out the same way like before.

### 1.5.2. Exponential distribution

## Exmaple

Suppose that the coal trains arrival times are exponential in form Then

$$
\begin{array}{lr}
\mathrm{f}(\mathrm{x})=\lambda \mathrm{e}^{-\lambda \mathrm{x}} & \mathrm{X}>=0 \text { and } \lambda>0 \\
\mathrm{~F}(\mathrm{x})=\int \lambda \mathrm{e}^{-\lambda \mathrm{x}} & \text { from0:X }
\end{array}
$$

$$
\mathrm{F}(\mathrm{x})=1-\mathrm{e}^{-\lambda x}
$$

And the mean value

$$
v x=1 / \lambda
$$




Generating exponentially distributed random numbers variants:
Can be done as follows:

$$
\begin{aligned}
& \mathrm{R}=12 *----------- \\
& \mathrm{X}=-(1 / \lambda) * \log (\mathrm{R})
\end{aligned}
$$

Where:

$$
\mathrm{X}=\text { variable random value (i.e. arrival time) }
$$

### 1.5.3. Log-Normal Distribution

The lognormal distributions are defined in the following case:
If say: $(Y=\log x)$, is normally distrusted(Bill shaped) in the range $-\infty<y<\infty$ then this ( x ) is said to be log-normal distributed.

Now the value of mean, standard deviation is calculated corresponding to the normalized function i.e. (y)

$$
v_{x}=E(x)=\exp ^{\left(\nu y+\frac{\sigma_{y}^{2}}{2}\right)}
$$

$$
\begin{aligned}
& \sigma_{x}=\exp \left(2 * v_{y}+\sigma_{y}^{2}\right) * \exp \left(\sigma_{y}^{2}-1\right) \\
& \mu_{y}=\ln \left(\mu_{x}\right)-1 / 2 \ln \left(\frac{\sigma_{x}}{\sigma_{y}}+1\right) \\
& \sigma_{y}^{2}=\ln \left(\sigma_{x}^{2} / \mu_{x}^{2}+1\right)
\end{aligned}
$$

Standard normal variant (z) can be defined as:

$$
\begin{aligned}
& z=\frac{\ln x-\mu y}{\sigma_{y}} \\
& x=\exp \left(\mu y+\sigma_{y} * z\right) \\
& x=\exp \left(\mu_{x}+\sigma_{y}\left[\left(\frac{k}{12}\right)^{\frac{-1}{2}}\left(\sum_{i=1}^{k} r_{i}-\frac{k}{2}\right)\right]\right. \\
& \text { where }: k=12 \\
& x=\exp \left[\mu_{y}+\sigma_{y}\left[\sum_{i=1}^{12}\left(r_{i}-6\right)\right]\right]
\end{aligned}
$$

## Example :

Mean=20, $\sigma_{x}^{2}=5, \Sigma r_{i}=4$ for 12 Random Number, generate the value coming from the log-normal distributions.
Answer

$$
\begin{aligned}
\mu_{y} & =\ln \left(\mu_{x}\right)-1 / 2\left[\sigma_{x}^{2} / \mu_{x}+1\right] \\
& =\ln (20)-1 / 2 \ln (5 / 20+1)=2.88 \\
\sigma y & =\sqrt{ } \ln \left[\sigma^{2} / \mu^{2}+1\right]= \\
& =\sqrt{ } \ln \left[5 /(20)^{2}+1\right]=0.111
\end{aligned}
$$

$$
\begin{aligned}
& X=\exp \left[\mu_{y}+\sigma_{y}\left[\Sigma r_{i}-6\right]\right] \\
& X=\exp [2.88+0.111[4-6] \\
& X=15.9
\end{aligned}
$$

### 1.6. Project scheduling by CPM - PERT

The method of planning any project is varied. The CPM (critical path method) and also PERT (project evaluation reviews techniques), will be discussed here.

For the project scheduling technique there are three important elements including the following:

## I-Planning

Break the project into distinct activates. Determine the time estimates of the activates. Construct a network diagram with each arc in the network representing an activity of the project.

## II-Scheduling

the objective is to construct a time chart showing the start and finish times for each activity. As will as it's relationship to other activities in the project. Schedule must show the critical activities that required special attention for usefully accomplished.

## III-Project Control

Network diagram and the time chart for making periodic progress reports. the network is updated periodically and analyzed if necessary to determine a new scheduling.

## 1 - Network diagram representation

Each activity is represented by an arrow with the head indicating the direction of the progress. Preceding relationships are specified by using events.

An event represents a point (node) in the time that signifies the completion of the same activities and beginning of new ones. So a direct arc and each event represent each activity represented by a node.

### 1.6.1. Rules for network construction

Rule (1) : Each activity is represented by one and only one arrow.
Rule (2) : No two activates can be identified by the same head and tail event


Rule (3) : to ensure the correct precedence relationships in the arrow diagram the following questions must be answered:
a- What activity must be completed immediately before this activity can start
b- What activity must follow this activity
c- What activities a must occur concurrently with this activity.

## Example

Construct a network diagram comprising activities $\mathrm{A}, \mathrm{B}, \mathrm{C} \ldots .$. . and L such that the following relationships are satisfied.

1- A, B and C, are first activities of the project can start simultaneously.
2- A and B precede D .
3- B precede $\mathrm{E}, \mathrm{F}$ and H
4- F and C precede G
5- E and H precede J and I .
6- C, D, F and J precede K
7- K precede L
8- I, G and $L$ are terminal activities of the project.


## Example

The following table represents the activates to build up housing project:

| Activity | Description | Immediate <br> Predecessor | Durations <br> (Day) |
| :---: | :---: | :---: | :---: |
| A | Clear site | - | 1 |
| B | Bringing utilities to site | - | 2 |
| C | Excavate | A | 1 |
| D | Pour foundation | C | 2 |
| E | Outside plumbing | $\mathrm{B}, \mathrm{C}$ | 6 |
| F | Frame house | D | 10 |
| G | Electric wiring | F | 3 |
| H | Lay floor | G | 1 |
| I | Lay roof | F | 1 |
| J | Inside plumbing | $\mathrm{E}, \mathrm{H}$ | 5 |
| K | Shingling | I | 2 |
| L | Outside sheathing insulation | $\mathrm{K}, \mathrm{J}$ | 1 |
| M | Install windows and outside doors | F | 2 |
| N | Brick work | $\mathrm{L}, \mathrm{M}$ | 4 |
| O | Insulate walls and ceiling | $\mathrm{G}, \mathrm{J}$ | 2 |
| P | Cover walls and ceiling | O | 2 |
| Q | Insulate roof | $\mathrm{I}, \mathrm{P}$ | 1 |
| R | Furnish interiors | P | 7 |
| S | Finish exteriors | $\mathrm{L}, \mathrm{N}$ | 7 |
| T | Landscape | S | 3 |

## Answer

The following diagram is the most probable for this project


Network diagram of the example

### 1.6.2. Critical Path Analysis

Suppose we have the following network diagram


The critical path is defined as follows:
It's the longest path through the project network time scheduling
There are two path estimating methods: forward pass and back ward pass depending in where we start estimating the path at node (0) or at (node final)

Let's try forward path method for the previous project:
The longest duration for an event is defined as (TE=Earliest time to finish.)


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[^0]:    Note: $\mathrm{T}_{\mathrm{i}}$ representing the Random Realization for failure occurrence in this system

